$\Leftrightarrow |x+1| = |x+2|^2 = (x+2)^2$

$\Leftrightarrow |x+1| = x^2+4x+4$

$\Leftrightarrow x^2+4x+4 = x+1$ ou $x^2+4x+4 = -x-1$

$\Leftrightarrow x^2+3x+3=0$ ou $x^2+5x+5=0$

$\Delta = 9-12 < 0$ | $\Delta = 25-20 = 5$

$S_1 = \varnothing$

$\Leftrightarrow x' = \dfrac{-5+\sqrt{5}}{2}, x'' = \dfrac{-5-\sqrt{5}}{2}$

$S = \left\{\dfrac{-5+\sqrt{5}}{2}, \dfrac{-5-\sqrt{5}}{2}\right\}$

g) $\ln|x+3| + \ln|x+5| = \ln 15$, $x \neq -5$ et $x \neq -3$

$\ln|(x+3)(x+5)| = \ln 15 \Leftrightarrow |(x+3)(x+5)| = 15$

$\Leftrightarrow \begin{cases} (x+3)(x+5) = -15 \\ (x+3)(x+5) = 15 \end{cases}$

$\Leftrightarrow \begin{cases} x^2+8x+15 = 15 & (1) \\ x^2+8x+15 = -15 & (2) \end{cases}$

(1) donne $x(x+8) = 0 \Leftrightarrow x = 0$ ou $x = -8$

(2) donne $x^2+8x+30 = 0$, on a $\Delta < 0$

D'où $S = \{0, -8\}$

Exercice 6

a) $\ln^2 x - \ln\dfrac{1}{x} - 2 = 0 \Rightarrow \ln^2 x + \ln x - 2 = 0$

$\Rightarrow (\ln x - 1)(\ln x + 2) = 0, x > 0$

$\Rightarrow \ln x = 1$ ou $\ln x = -2$

$\Rightarrow x = e$ ou $x = e^{-2} = \dfrac{1}{e^2}$

$S = \left\{\dfrac{1}{e^2}, e\right\}$

b) $1 + \ln x = \dfrac{6}{\ln x}$, $x > 0$ et $x \neq 1$

$\Rightarrow \ln x + \ln^2 x - 6 = 0$

Posons $t = \ln x$

$\Rightarrow \begin{cases} t^2+t-6=0 \\ t = \ln x \end{cases}$

$t^2+t-6=0, \Delta = 1+24 = 25$

$t' = \dfrac{-1+5}{2} = 2$, $t'' = \dfrac{-1-5}{2} = -3$

Donc

$\ln x = 2 \Rightarrow x = e^2$

$\ln x = -3 \Rightarrow x = e^{-3}$

$S = \left\{\dfrac{1}{e^3}, e^2\right\}$.

c) $\ln x - 2\ln(x-4) + \ln 2 = 0$, *existe* pour $x > 4$

$\underbrace{\ln x + \ln 2}_{\downarrow} - \underbrace{2\ln(x-4)}_{\downarrow} = 0$

$\Rightarrow \ln 2x - \ln\left((x-4)^2\right) = 0 \Rightarrow 2x = (x-4)^2$

$\Rightarrow 2x = x^2 - 8x + 16 \Rightarrow x^2 - 10x + 16 = 0$

$\Delta' = 25-16 = 9 > 0$

$x' = 5+3 = 8 > 4$ ou $x'' = 5-3 = 2 < 4$

Donc $S = \{8\}$

d) $\ln|x+1| < 1$, $x \neq -1$

$\Rightarrow |x+1| < e \Leftrightarrow -e < x+1 < e$

$\Rightarrow -e-1 < x < e-1$, $x \neq -1$

Donc $S =]-e-1, e-1[\setminus \{-1\}$

e) $\ln|x| < -\ln|3x+2|$ (E)

$x \in \mathbb{R}^* \setminus \left\{-\frac{2}{3}\right\}$

$(E) \Leftrightarrow \ln|x| + \ln|3x+2| < 0$

$\Leftrightarrow \ln|x(3x+2)| < 0$

$\Leftrightarrow |x(3x+2)| < 1$

$\Leftrightarrow -1 < x(3x+2) < 1$

$\Leftrightarrow 3x^2+2x-1 < 0$ ou $3x^2+2x+1 > 0$

$a-b+c=0$ | $\Delta = 4-12 < 0$

$x' = -1, x'' = \dfrac{1}{3}$

$S_2 = \mathbb{R}^* \setminus \left\{\dfrac{-2}{3}\right\}$

$\mathbb{R}$ *T.signe*

x	$-\infty$		-1		$\frac{1}{3}$		$+\infty$
Signe de $3x^2+2x-1$	$+$		0	$-$	0		$+$